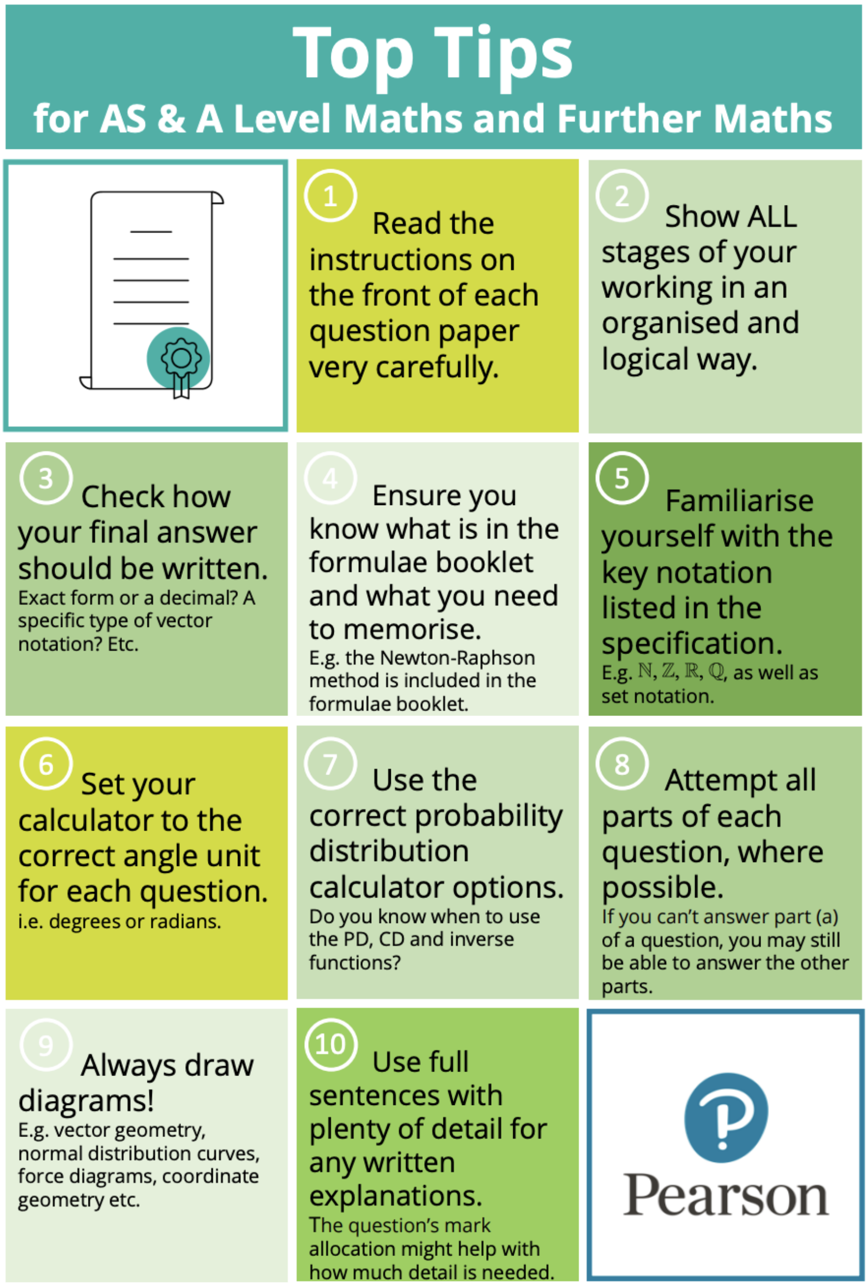
Revision lesson on differential equations

you CHAN do it



Question:	1	2	3	4	5	6
Points:	5	5	8	13	5	9
Score:						

Question:	7	8	9	10		Total
Points:	15	15	12	14		101
Score:						

1.

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x}{y}$$

(a) find a general solution in the form $y = f(x)$

(3)

(b) find a particular solution, given that when $x = 1$, $y = 2$

(2)

2.

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y}{x}$$

(a) find a general solution in the form $y = f(x)$

(3)

(b) find a particular solution, given that when $x = 1$, $y = 2$

(2)

Working Space

3. (a) Find

$$\int \frac{9x+6}{x}\, \mathrm{d}x, \quad x > 0$$

(2)

(b) Given that $y = 8$ at $x = 1$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(9x+6)y^{\frac{1}{3}}}{x}$$

giving your answer in the form $y^2 = g(x)$.

(6)

Working Space

4. Liquid is pouring into a container at a constant rate of 20 $\text{cm}^3\ \text{s}^{-1}$ and is leaking out at a rate proportional to the volume of liquid already in the container.

(a) Explain why, at time t seconds, the volume, V cm^3, of liquid in the container satisfies the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = 20 - kV,$$

where k is a positive constant.

(2)

The container is initially empty.

(b) By solving the differential equation, show that

$$V = A + B\mathrm{e}^{-kt},$$

giving the values of A and B in terms of k.

(6)

Given also that $\frac{\mathrm{d}V}{\mathrm{d}t} = 10$ when $t = 5$,

(c) find the volume of liquid in the container at 10 s after the start.

(5)

(a) $\frac{\mathrm{d}V}{\mathrm{d}t}$ [illegible]

$-kV$: k [illegible]

giving $\frac{\mathrm{d}V}{\mathrm{d}t} = 20 - kV$ * [illegible]

Working Space

5. Water is flowing into a large container and is leaking from a hole at the base of the container.

At time t seconds after the water starts to flow, the volume, V cm^3, of water in the container is modelled by the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = 300 - kV$$

where k is a constant.

(a) Solve the differential equation to show that, according to the model,

$$V = \frac{300}{k} + A\mathrm{e}^{-kt}$$

where A is a constant.

(5)

Skipping Subsequent Parts in the original question this time, but well done on getting this far, year 13s!

Working Space

6. A bottle of water is put into a refrigerator. The temperature inside the refrigerator remains constant at 3°C and t minutes after the bottle is placed in the refrigerator the temperature of the water in the bottle is θ°C.

The rate of change of the temperature of the water in the bottle is modelled by the differential equation,

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{(3 - \theta)}{125}$$

(a) By solving the differential equation, show that,

$$\theta = A\mathrm{e}^{-0.008t} + 3$$

where A is a constant.

(4)

Given that the temperature of the water in the bottle when it was put in the refrigerator was 16°C,

(b) find the time taken for the temperature of the water in the bottle to fall to 10°C, giving your answer to the nearest minute.

(5)

Working Space

Spot the Difference!

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n assist with filling in this form.

11. (a) Given $0 \leqslant h < 25$, use the substitution $u = 5 - \sqrt{h}$ to show that

$$\int \frac{\mathrm{d}h}{5 - \sqrt{h}} = -10\ln\left(5 - \sqrt{h}\right) - 2\sqrt{h} + k$$

where k is a constant.

(6)

A team of scientists is studying a species of tree.

The rate of change in height of a tree of this species is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{t^{0.2}\left(5 - \sqrt{h}\right)}{5}$$

where h is the height of the tree in metres and t is the time in years after the tree is planted.

One of these trees is 2 metres high when it is planted.

(b) Use integration to calculate the time it would take for this tree to reach a height of 15 metres, giving your answer to one decimal place.

(7)

(c) Hence calculate the rate of change in height of this tree when its height is 15 metres. Write your answer in centimetres per year to the nearest centimetre.

(1)

January 2014 (IAL) QP - C34 Edexcel 28 / 44 100%

PhysicsAndMathsTutor.com January 2014 (IAL)

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9. (a) Use the substitution $u = 4 - \sqrt{x}$ to find

$$\int \frac{\mathrm{d}x}{4 - \sqrt{x}}$$

(6)

A team of scientists is studying a species of slow growing tree.

The rate of change in height of a tree in this species is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{4 - \sqrt{h}}{20}$$

where h is the height in metres and t is the time measured in years after the tree is planted.

(b) Find the range in values of h for which the height of a tree in this species is increasing.

(2)

(c) Given that one of these trees is 1 metre high when it is planted, calculate the time it would take to reach a height of 10 metres. Write your answer to 3 significant figures.

(7)

14. (a) Use the substitution $u = 4 - \sqrt{h}$ to show that

$$\int \frac{\mathrm{d}h}{4-\sqrt{h}} = -8\ln\left|4-\sqrt{h}\right| - 2\sqrt{h} + k$$

where k is a constant

(6)

A team of scientists is studying a species of slow growing tree.

The rate of change in height of a tree in this species is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{t^{0.25}\left(4-\sqrt{h}\right)}{20}$$

where h is the height in metres and t is the time, measured in years, after the tree is planted.

(b) Find, according to the model, the range in heights of trees in this species.

(2)

One of these trees is one metre high when it is first planted.

According to the model,

(c) calculate the time this tree would take to reach a height of 12 metres, giving your answer to 3 significant figures.

(7)

7.

January 2014 (IAL) QP - C34 Edexcel 28 / 44 100%

Leave blank

9. (a) Use the substitution $u = 4 - \sqrt{x}$ to find

$$\int \frac{\mathrm{d}x}{4 - \sqrt{x}}$$

(6)

A team of scientists is studying a species of slow growing tree.

The rate of change in height of a tree in this species is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{4 - \sqrt{h}}{20}$$

where h is the height in metres and t is the time measured in years after the tree is planted.

(b) Find the range in values of h for which the height of a tree in this species is increasing.
(2)

(c) Given that one of these trees is 1 metre high when it is planted, calculate the time it would take to reach a height of 10 metres. Write your answer to 3 significant figures.
(7)

Working Space

8.

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11. (a) Given $0 \leqslant h < 25$, use the substitution $u = 5 - \sqrt{h}$ to show that

$$\int \frac{\mathrm{d}h}{5 - \sqrt{h}} = -10\ln\left(5 - \sqrt{h}\right) - 2\sqrt{h} + k$$

where k is a constant.

(6)

A team of scientists is studying a species of tree.

The rate of change in height of a tree of this species is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{t^{0.2}\left(5 - \sqrt{h}\right)}{5}$$

where h is the height of the tree in metres and t is the time in years after the tree is planted.

One of these trees is 2 metres high when it is planted.

(b) Use integration to calculate the time it would take for this tree to reach a height of 15 metres, giving your answer to one decimal place.

(7)

(c) Hence calculate the rate of change in height of this tree when its height is 15 metres. Write your answer in centimetres per year to the nearest centimetre.

(1)

Working Space

9. (a) Express $\dfrac{1}{P(5-P)}$ in partial fractions.

(3)

A team of conservationists is studying the population of meerkats on a nature reserve. The population is modelled by the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1}{15}P(5-P), \quad t \geqslant 0$$

where P, in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that when $t = 0, P = 1$,

(b) solve the differential equation, giving your answer in the form,

$$P = \frac{a}{b + c\mathrm{e}^{-\frac{1}{3}t}}$$

where a, b and c are integers.

(8)

(c) Hence show that the population cannot exceed 5000

(1)

Working Space

10. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Write $\dfrac{1}{(H-5)(H+3)}$ in partial fraction form.

(3)

The depth of water in a storage tank is being monitored.

The depth of water in the tank, H metres, is modelled by the differential equation

$$\frac{\mathrm{d}H}{\mathrm{d}t} = -\frac{(H-5)(H+3)}{40}$$

where t is the time, in days, from when monitoring began.

Given that the initial depth of water in the tank was 13 m,

(b) solve the differential equation to show that

$$H = \frac{10 + 3\,\mathrm{e}^{-0.2t}}{2 - \mathrm{e}^{-0.2t}}$$

(7)

(c) Hence find the time taken for the depth of water in the tank to fall to 8 m.

(3)

According to the model, the depth of water in the tank will eventually fall to k metres.

(d) State the value of the constant k.

(1)

Working Space

Extra Working Space

Total for paper is 101 marks