Year 13 Review
Revision: Parametrics + Connected Rates of Change

Contents

Question	Points	Score
1	8	
2	12	
3	16	
4	12	
5	10	
6	9	
7	11	
8	10	
9	12	
Total:	100	

1 Parametrics

1.1 Parametric Differentiation

1.1.1 CIE - 9709-21-June23 Q5

1.

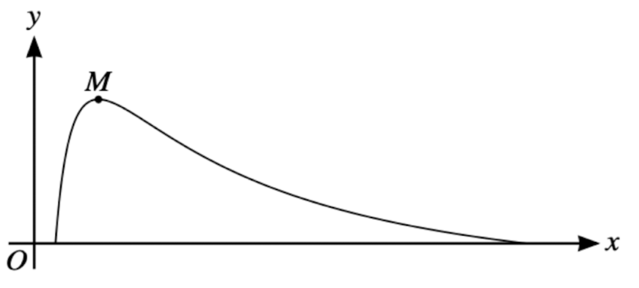

Figure 1

Figure 1 shows the curve with parametric equations

$$x = 4e^{2t}, \quad y = 5e^{-t}\cos 2t \quad -\frac{1}{4}\pi \leqslant t \leqslant \frac{1}{4}\pi$$

The curve has a maximum point M.

(a) Find an expression for $\dfrac{dy}{dx}$ in terms of t.

(3)

(b) Find the coordinates of M, giving each coordinate correct to 3 significant figures.

(5)

Question 1 continued

(Total for Question 1 is 8 marks)

1.1.2 OCR(A) 2022 Paper 1, Q12

2. A curve has parametric equations

$$x = \frac{1}{t},\ y = 2t$$

The point P is $\left(\frac{1}{p}, 2p\right)$.

(a) Show that the equation of the tangent at P can be written as $y = -2p^2x + 4p$.

(4)

The tangent to this curve at P crosses the x-axis at the point A and the normal to this curve at P crosses the x-axis at the point B.

(b) Show that the ratio $PA : PB$ is $1 : 2p^2$.

(8)

Question 2 continued

(Total for Question 2 is 12 marks)

1.2 All of Parametrics

1.2.1 Q14 WMA02/01, June 2017

3.

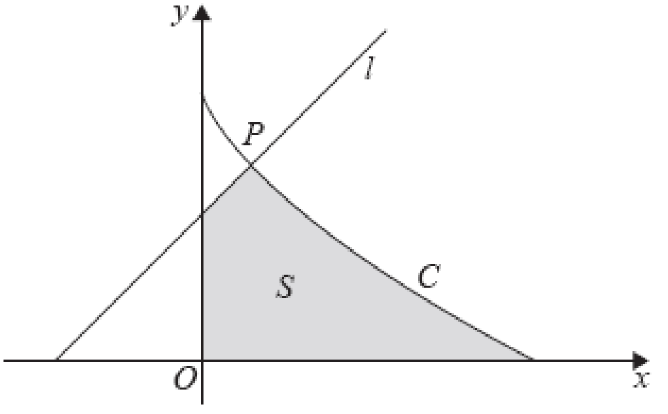

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 8\cos^3\theta, \quad y = 6\sin^2\theta, \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

Given that the point P lies on C and has parameter $\theta = \dfrac{\pi}{3}$

(a) find the coordinates of P.

(2)

The line l is the normal to C at P.

(b) Show that an equation of l is $y = x + 3.5$

(5)

The finite region S, shown shaded in Figure 2 , is bounded by the curve C, the line l, the y-axis and the x-axis.

(c) Show that the area of S is given by

$$4 + 144\int_0^{\frac{\pi}{3}} \left(\sin\theta\cos^2\theta - \sin\theta\cos^4\theta\right) d\theta$$

(6)

(d) Hence, by integration, find the exact area of S.

(3)

(Solutions based ***entirely*** *on graphical or numerical methods are not acceptable.)*

Question 3 continued

(Total for Question 3 is 16 marks)

1.2.2 Textbook Mixed Ex 11, Q29

4.

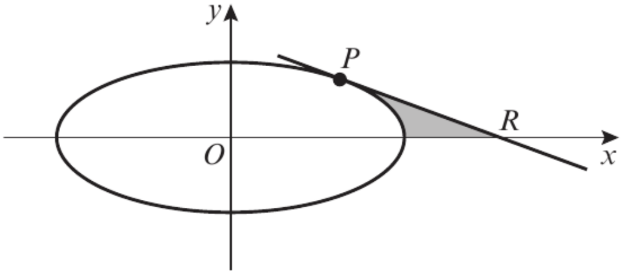

Figure 3

Figure 3 shows the curve with parametric equations

$$x = 5\cos\theta, y = 4\sin\theta, \qquad 0 \leqslant \theta \leqslant 2\pi$$

(a) Find the gradient of the curve at the point P at which $\theta = \dfrac{\pi}{4}$

(3)

(b) Find an equation of the tangent to the curve at the point P.

(3)

(c) Find the exact area of the shaded region bounded by the tangent PR, the curve and the x-axis.

(6)

Question 4 continued

(Total for Question 4 is 12 marks)

2 Connected Rates of Change - Differential Equations

2.1 IAL C34 - Q10 WMA02/01, Jan 2019 - Connected Rates of Change

5.

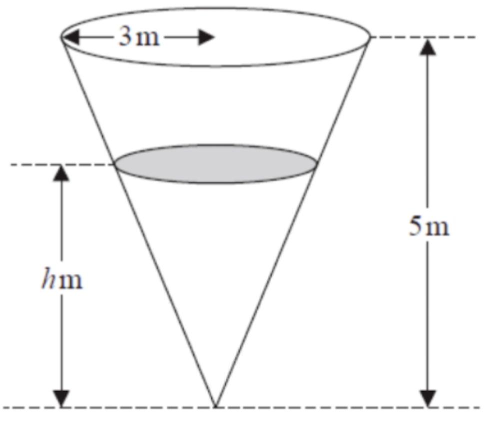

Figure 4

Figure 4 shows a container in the shape of an inverted right circular cone which contains some water.

The cone has an internal radius of 3 m and a vertical height of 5 m, as shown in Figure 4.

At time t seconds, the height of the water is h metres, the volume of the water is V m^3 and water is leaking from a hole in the bottom of the container at a constant rate of 0.02 m^3 s^{-1}

[The volume of a cone of radius r and height h is $\frac{1}{3}\pi r^2 h$.]

(a) Show that, while the water is leaking,

$$h^2\frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{1}{k\pi}$$

where k is a constant to be found.

(5)

Given that the container is initially full of water,

(b) express h in terms of t.

(3)

(c) Find the time taken for the container to empty, giving your answer to the nearest minute.

(2)

2.2 Bonus Q: Solomon C4 Differentiation WorkSheet D Q6

6.

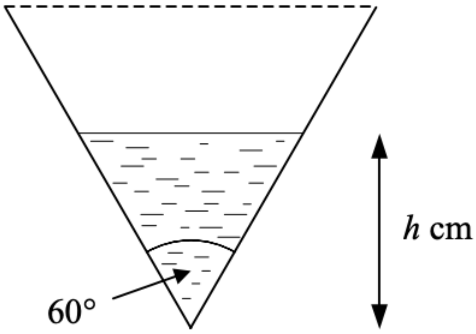

Figure 5

Figure 5 shows the cross-section of a right-circular paper cone being used as a filter funnel. The volume of liquid in the funnel is V cm^3 when the depth of the liquid is h cm.

Given that the angle between the sides of the funnel in the cross-section is 60° as shown,

(a) show that $V = \frac{1}{9}\pi h^3$.

Given also that at time t seconds after liquid is put in the funnel

$$V = 600\,\mathrm{e}^{-0.0005t},$$

(b) show that after two minutes, the depth of liquid in the funnel is approximately 11.7 cm

(c) find the rate at which the depth of liquid is decreasing after two minutes.

2.3 Edexcel Mock Set 2, Paper 1 Q13 - Connected Rates of Change

7.

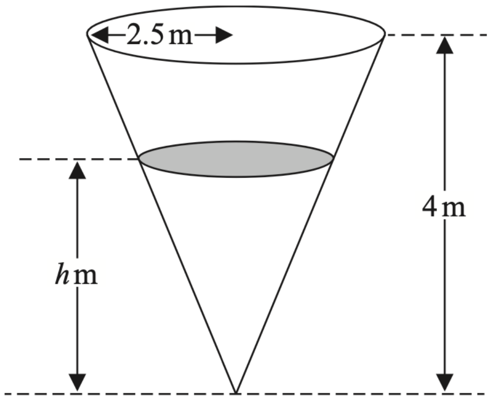

Figure 6

[The volume of a cone of base radius r and height h is $\frac{1}{3}\pi r^2 h$*]*

Figure 6 shows a container in the shape of an inverted right circular cone which contains some water.

The cone has an internal base radius of 2.5 m and a vertical height of 4 m.

At time t seconds

- the height of the water is h m
- the volume of the water is V m^3
- the water is modelled as leaking from a hole at the bottom of the container at a rate of

$$\left(\frac{\pi}{512}\sqrt{h}\right) \text{ m}^3 \text{ s}^{-1}$$

(a) Show that, while the water is leaking

$$h^{\frac{3}{2}}\frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{1}{200}$$

(5)

Given that the container was initially full of water,

(b) find an equation, in terms of h and t, to model this situation.

(3)

It takes approximately 43 minutes for the container to empty.

(c) Use this information to comment on the suitability of this model.

(3)

2.4 IAL P4 - Q09 WMA14/01, Oct 2021 - Connected Rates of Change

8.

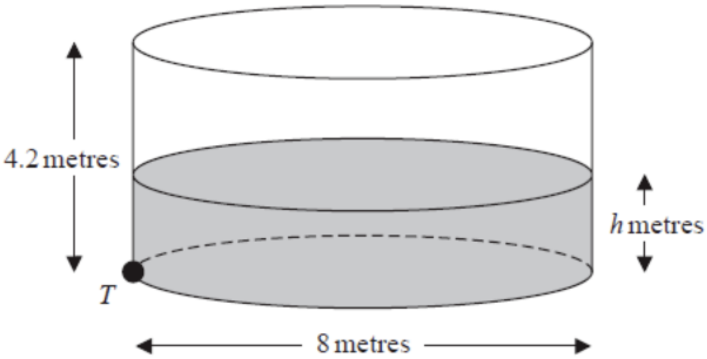

Figure 7

Figure 7 shows a cylindrical tank that contains some water.
The tank has an internal diameter of 8 m and an internal height of 4.2 m.
Water is flowing into the tank at a constant rate of (0.6π) m^3 per minute.
There is a tap at point T at the bottom of the tank.
At time t minutes after the tap has been opened,

- the depth of the water is h metres
- the water is leaving the tank at a rate of $(0.15\pi h)$ m^3 per minute

(a) Show that

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{12 - 3h}{320}$$

(4)

Given that the depth of the water in the tank is 0.5 m when the tap is opened,

(b) find the time taken for the depth of water in the tank to reach 3.5 m.

(6)

Question 8 continued

(Total for Question 8 is 10 marks)

2.5 IAL C34 - Q14 WMA02/01, Jan 2018 - Connected Rates of Change

9. The volume of a spherical balloon of radius r cm is V cm^3, where $V = \dfrac{4}{3}\pi r^3$

(a) Find $\dfrac{\mathrm{d}V}{\mathrm{d}r}$

(1)

The volume of the balloon increases with time t seconds according to the formula

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{9000\pi}{(t+81)^{\frac{5}{4}}} \qquad t \geqslant 0$$

(b) Show that

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{k}{r^n(t+81)^{\frac{5}{4}}} \qquad t \geqslant 0$$

where k and n are constants to be found.

(2)

Initially, the radius of the balloon is 3 cm.

(c) Using the values of k and n found in part (b), solve the differential equation

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{k}{r^n(t+81)^{\frac{5}{4}}} \qquad t \geqslant 0$$

to obtain a formula for r in terms of t.

(6)

(d) Hence find the radius of the balloon when $t = 175$, giving your answer to 3 significant figures.

(1)

(e) Find the rate of increase of the radius of the balloon when $t = 175$. Give your answer to 3 significant figures.

(2)

Question 9 continued

(Total for Question 9 is 12 marks)

Total for paper is 100 marks