Year 13 Pre-Mock Applied (Statistics) Set 01

- Advised to print in "A3-booklets", this will allow all questions to be on the left hand side.
- You can also print in A4, double-sided, and two staples on the left
- If instead you print in 2-in-1 settings, first print the second page up to the last page, then print the cover page separately (to allow all questions on the left)

This exam paper has 5 questions, for a total of 50 marks.

Question	Marks	Score
1	11	
2	7	
3	9	
4	10	
5	13	
Total:	50	

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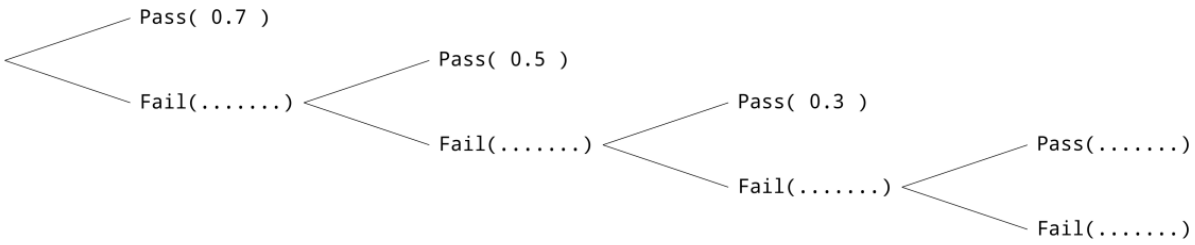
Last updated: 20th February 2023

1. A training agency awards a certificate to each student who passes a test while completing a course.

Students failing the test will attempt the test again up to 3 more times, and, if they pass the test, will be awarded a certificate.

The probability of passing the test at the first attempt is 0.7, but the probability of passing reduces by 0.2 at each attempt.

(a) Complete the tree diagram below to show this information.



(2)

A student who completed the course is selected at random.

(b) Find the probability that the student was awarded a certificate.

(2)

(c) Given that the student was awarded a certificate, find the probability that the student passed on the first or second attempt.

(3)

The training agency decides to alter the test taken by the students while completing the course, but will not allow more than 2 attempts.

The agency requires the probability of passing the test at the first attempt to be p, and the probability of passing the test at the second attempt to be $(p - 0.2)$.

The percentage of students who complete the course and are awarded a certificate is to be 95%.

(d) Show that p satisfies the equation

$$p^2 - 2.2p + 1.15 = 0$$

(3)

(e) Hence, state the value of p.

(1)

Question 1 continued

Question 1 continued

Question 1 continued

(Total for Question 1 is 11 marks)

2. A researcher, Chinemerem, is studying the birth weights of babies. A random sample of 98 babies was taken and their birth weights, w kg are summarised in the table below.

Birth weight (w kg)	Frequency (f)	Birth weight midpoint (x)
$1.50 \leq w < 2.50$	16	2.00
$2.50 \leq w < 3.00$	24	2.75
$3.00 \leq w < 3.50$	32	3.25
$3.50 \leq w < 4.00$	14	3.75
$4.00 \leq w < 5.50$	12	4.75

(You may use $\sum fx = 311.5$ and $\sum fx^2 = 1051.125$)

(a) Use linear interpolation to estimate the lower quartile of the birth weights of the 98 babies.

(2)

Chinemerem estimated the median to be 3.14 kg.

(b) Find an estimate for

(i) the mean birth weight

(ii) the standard deviation of the birth weights.

(3)

Chinemerem read that birth weights should be approximately normally distributed and decides to split the class $3.00 \leq w < 3.50$
The frequency for $3.00 \leq w < 3.25$ is 9 and the frequency for $3.25 \leq w < 3.50$ is 23

(c) (i) State, giving a reason, what the effect would be on the estimate of the median.

(ii) Without carrying out any further calculations state, giving a reason, what the effect of this change would be on the estimate of the mean.

(2)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 7 marks)

3. An ornithologist, Ben, has data on the number of breeding pairs of bald eagles for selected years from 1963 to 2000.
Ben believes that the number of breeding pairs of bald eagles can be modelled by

$$x = a(b^T) \qquad \text{(I)}$$

where T is the number of years since 1950, x is the number of breeding pairs of bald eagles and a and b are constants.

(a) Show that the equation can be written in the form $\log_{10} x = mT + c$

(3)

Ben codes the data using the coding $Y = \log_{10} x$ and $X = T$.
He then obtains the model $Y = 0.0348X + 2.0827$

(b) Determine the values of a and b.

(3)

Ben wants to use the model (I) to estimate the number of breeding pairs of bald eagles in 2020.

(c) Explain why this would not be appropriate.

(1)

There is no record of the number of breeding pairs of bald eagles for 1970.

(d) Use the model (I) to estimate how many breeding pairs of bald eagles there would have been in 1970.

(2)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 9 marks)

4.

Figure 1

Hook-a-duck is a traditional fairground stall game, as shown in Figure 1.
In a game of *Hook-a-duck*, players use a hook on the end of a piece of string to catch a plastic duck floating in a pool.
Each duck looks the same but has a number written on the bottom: 1, 2, 3 or 4
There are 20 ducks in the pool, 5 with each number.
Each time a duck is hooked it is put back in the pool ready for the next player.

(a) State the distribution which may be used to model the number on a duck hooked by a randomly selected player.

(1)

Using the distribution suggested in part (a), calculate the probability that of the first 3 ducks hooked,

(b) all 3 have the same number,

(2)

(c) exactly 2 of them have the same number.

(3)

During the day 200 ducks are hooked, the random variable X is defined as the number of ducks out of 200 that have an **even** number.

(d) State an appropriate distribution which may be used to model the random variable X.

(1)

(e) State a necessary assumption about hooking the ducks which must be true for this distribution to be a good model.

(1)

(f) Use this model to calculate the probability that, out of the 200 ducks hooked that day, at least 110 ducks have even numbers.

(2)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 10 marks)

5. Eve's machine puts liquid into bottles of medicine. The amount of liquid put into each bottle, D ml, follows a normal distribution with mean 28 ml.

Given that 5% of the bottles contain less than 27.29 ml,

(a) find, to 2 decimal places, the value of k such that $\mathrm{P}(27.29 < D < k) = 0.55$.

(5)

A random sample of 200 bottles is taken.

(b) Using a normal approximation, find the probability that fewer than half of these bottles contain between 27.29 ml and k ml.

(3)

Eve adjusted the machine so that the standard deviation of the liquid put in the bottles is now 0.7 ml.
Following the adjustments, Eve's assistant believes that the mean amount of liquid put in each bottle is less than 28 ml. The assistant takes a random sample of 20 bottles and finds the mean amount of liquid to be 27.72 ml.

(c) Showing all your working, test the assistant's belief at the 5% level of significance.

(5)

Question 5 continued

Question 5 continued

(Total for Question 5 is 13 marks)

Total for paper is 50 marks