Cross-topic linkage
Factor Theorem, Graphs and Transformations

Wednesday 16th October, 2024

Question 1

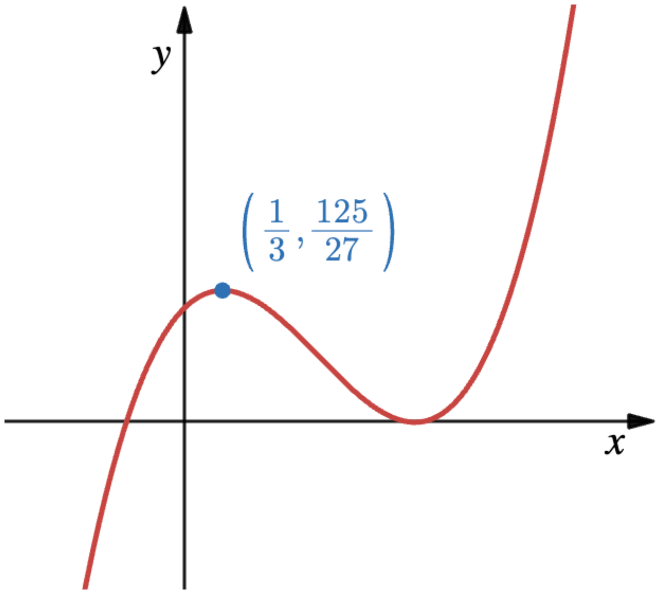

Figure 1: `https://www.desmos.com/calculator/vll0qkttz5`

Figure 1 shows a the curve C_1 with equation $y = f(x)$ where

$$f(x) = (x-2)^2(2x+1), \quad x \in \mathbb{R}$$

The maximum turning point at the point marked P.

A second curve C_2 has equation $y = f(x+1)$.

(a) Write down an equation of the curve C_2.

You may leave your equation in a factorised form.

(1)

(b) Find the coordinates of the point where the curve C_2 meets the y-axis.

(2)

(c) Write down the coordinates of the two turning points on the curve C_2.

(2)

(d) Sketch the curve C_2, with equation $y = f(x+1)$, giving the coordinates of the points where the curve crosses or touches the x-axis.

(3)

(Q14 WMA01/01, June 2018 adapted)

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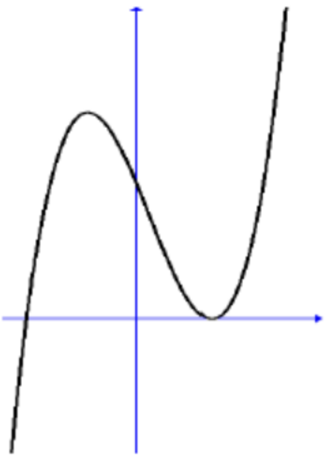
(Q14 WMA01/01, June 2018 adapted)

(b)	$y=(x-1)^2(2x+3)$	B1 [1]
(c)	When $x=0$, $y=3$	M1 A1 [2]
(d)	(1, 0) and $\left(-\frac{2}{3}, "\frac{125}{27}"\right)$	M1 A1ft [2]
(e)	 M1: Shape same as before, +ve cubic, but moved. Don't be overly concerned about the position of the maximum point. A1: Shape same as before but moved to the **left** (maximum must be in second quadrant and minimum on +ve x - axis) and graph lies in three quadrants A1: (1,0) and (-1.5,0) or marked on the x axis as 1 and -1.5	M1 A1 A1 [3]

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Question 2

$$f(x) = 3x^3 + 3x^2 + cx + 12, \text{ where } c \text{ is a constant}$$

Given that $(x+3)$ is a factor of $f(x)$,

(a) show that $c = -14$.

(2)

(b) Write $f(x)$ in the form

$$f(x) = (x+3)Q(x)$$

where $Q(x)$ is a quadratic function.

(2)

(c) Use the answer to part (b) to prove that the equation $f(x) = 0$ has only one real solution.

(2)

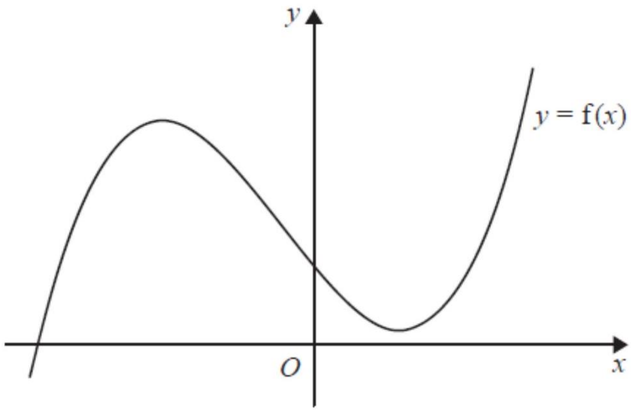


Figure 2

Figure 2 shows a sketch of the curve with equation $y = f(x), x \in \mathbb{R}$.

On separate diagrams sketch the curve with equation

(d) (i) $y = f(3x)$

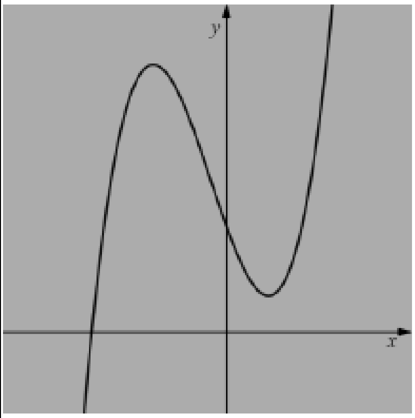
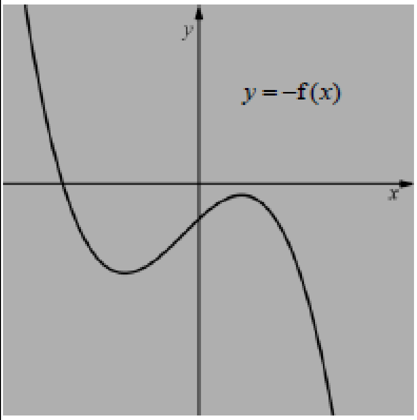
(ii) $y = -f(x)$

On each diagram show clearly the coordinates of the points where the curve crosses the coordinate axes.

(4)

(Q13 WMA01/01, Jan 2019)

((Q13 WMA01/01, Jan 2019))

Question Number	Scheme	Marks
(a)	Attempts $f(\pm 3)=0$	M1
	$-81+27-3c+12=0 \Rightarrow 3c=-42 \Rightarrow c=-14$	A1*
		(2)
(b)	$3x^3+3x^2-14x+12=(x+3)(3x^2-6x+4)$	M1 A1
		(2)
(c)	Attempts "b^2-4ac" for their $(3x^2-6x+4)$	M1
	$b^2-4ac=-12<0 \Rightarrow (3x^2-6x+4)$ has no roots and hence $f(x)=0$ has 1 root $(=-3)$	A1
		(2)
(d)(i)	 shape and position with x intercept at -1 or -9	B1
	Intercepts of $(-1,0)$ and $(0,12)$	B1
(d)(ii)	 Shape and position	B1
	Intercepts of $(-3,0)$ and $(0,-12)$ only	B1
		(4)
		(10 marks)

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Question 3

$$f(x) = -4x^3 + 16x^2 - 13x + 3$$

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$.

(2)

(b) Hence fully factorise $f(x)$.

(4)

(c) Sketch a graph of $y = f(x)$, showing clearly the coordinates of each point where the curve crosses the coordinate axes.

(3)

(d) Use your answer to part (c) and the sketch to deduce the set of values of x for which $f(x) \leq 0$

(2)

(Q05 WMA01/01, June 2017 adapted)

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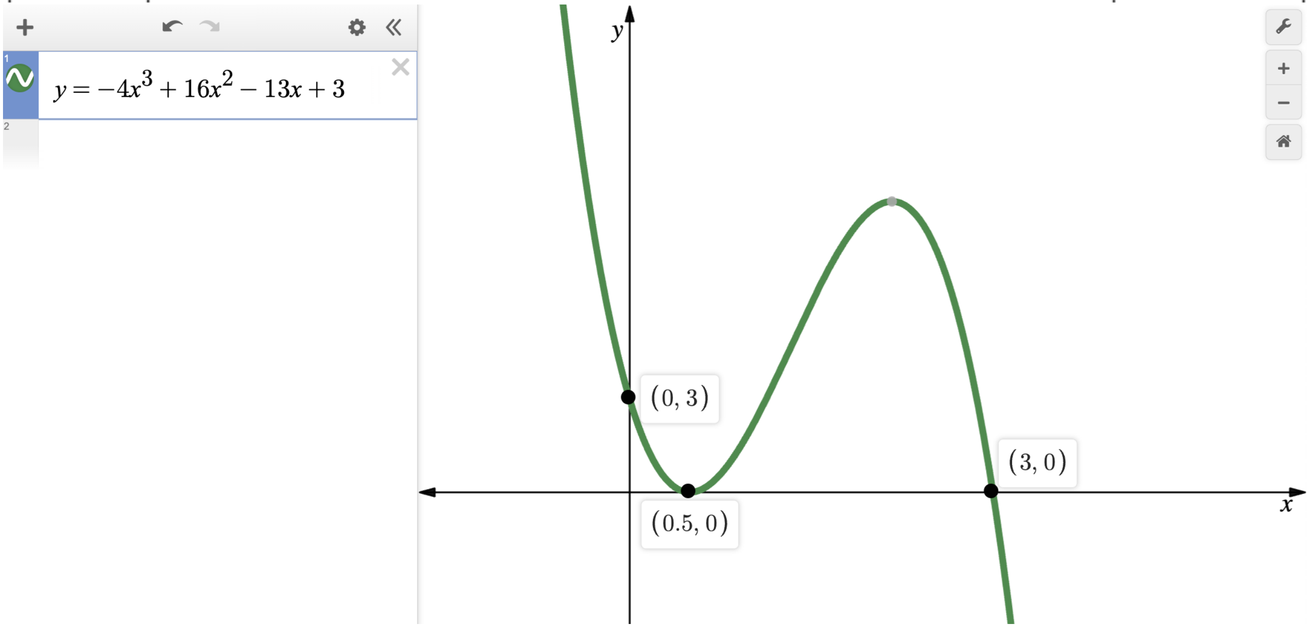
(Q05 WMA01/01, June 2017)

(b)	Attempts $f(\pm 3) = -4\times(\pm 3)^3 + 16\times(\pm 3)^2 - 13\times(\pm 3) + 3$	M1
	Remainder $= 0 \Rightarrow (x-3)$ is a factor	A1*
		(2)
(c)	Divides their f(x) by $(x-3)$ to get the quadratic factor	M1
	$\left(-4x^2+4x-1\right)$	A1
	$f(x) = (x-3)\times -(2x-1)(2x-1) = (3-x)(2x-1)^2$	dM1A1
		(4)
(d)	$f(x) \leqslant 0 \Rightarrow (3-x)(2x-1)^2 \leqslant 0$	
	$x = \frac{1}{2},\ x \geqslant 3$	B1,B1
		(2)



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Question 4

The curve C_1 has equation $y = f(x)$ where

$$f(x) = \left(x^2 - 4\right)(x - 3)$$

(a) Sketch a graph of C_1 showing clearly the coordinates of each point where the curve crosses the coordinate axes.

(3)

The finite region R is bounded by C_1 and the x-axis.

Given that R lies above the x-axis,

(b) Use inequality to define the region R

(2)

A second curve C_2 has equation $y = f(2x)$.

(c) (i) Write down an equation of C_2

(You may leave your equation in a factorised form.)

(ii) Describe geometrically the transformation that maps C_1 onto C_2

(Q11 WMA01/01, Oct 2019)

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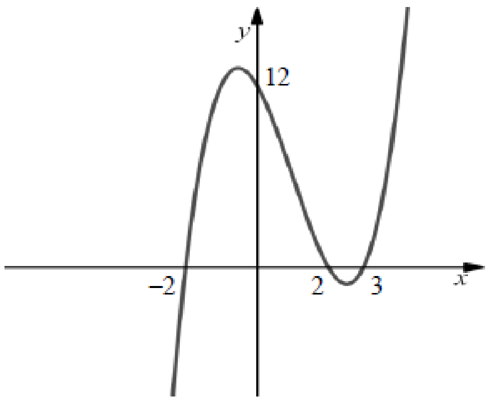
(Q11 WMA01/01, Oct 2019)

Question Number	Scheme		Marks
(a)		Shape: A positive cubic shape that crosses/touches the x-axis at least once. Allow the "ends" to turn back slightly as long as they do not tend to the horizontal or form extra turning points.	B1
		Intercepts: Allow for a y-intercept of 12 or x-intercepts of −2, 2 and 3 (See note below)	B1
		Correct shape with correct intercepts with a minimum in quadrant 4 and a maximum in quadrant 1 or quadrant 2 or at (0, 12). Allow the curve to stop at (−2, 0)	B1
	For the intercepts, allow them to be marked as shown in the diagram and also as e.g. (0, 12), (−2, 0), (2, 0), (3, 0) and allow the coordinates as (12, 0) etc. as long as they are marked in the correct places. If the coordinates are not on the diagram then they must be the right way round and correspond with the sketch. The sketch takes precedence if there is any ambiguity.		
	Note: If the sketch consists of 3 straight line segments but is otherwise correct award 110		
			[3]

Question Number	Scheme			Marks
(c)(i)	$(y=)(4x^2-4)(2x-3)$ Allow any equivalent correct expressions e.g. $(2x)^3-3(2x)^2-4(2x)+12,\ (2x-2)(2x+2)(2x-3)$ and "$y=$" not required. **isw once a correct expression is seen**			B1
(ii)	**In (c) part (ii), mark positively where possible** **Note that strictly speaking, a stretch requires an invariant line but we are not insisting that candidates refer to an invariant line here**			
	1. **Examples** Stretch/Contract/Shrink/ Compress/Enlarge/ Smaller/Thinner/ Contracted (Any idea of size change)	2. **Examples** Scale factor 0.5/Divides by 2	3. **Examples** Parallel to/on/at the x-axis/ Horizontally	M1A1
	M1: For any 2 of the above A1: For all of the above			
	<u>**Special Case: Covers 2 & 3 above**</u> x (values) divided by 2 (halved) on its own scores M1A0 - must reference x and halving e.g. accept for this special case • x halved • multiply x by ½			
				[3]

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