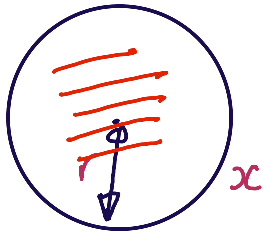
Mini Test 02 - Coordinate Geometry / Graphs

Bonus fun fact question:
Given that a shape has perimeter x cm, what is the largest possible area of this shape ? **[0.5 marks]**

$2\pi r = x$

$r = \frac{x}{2\pi}$



$\pi r^2 = A$

$\pi\left(\frac{x}{2\pi}\right)^2 = A$

$A = \frac{\pi x^2}{4\pi^2}$

$A = \frac{x^2}{4\pi}$ //

Question 1

$(x-4)^2+(y-2)^2=r^2$

$\sqrt{(4-1)^2+(2-0)^2} = r$

$\sqrt{9+4} = r$

$\sqrt{13} = r$

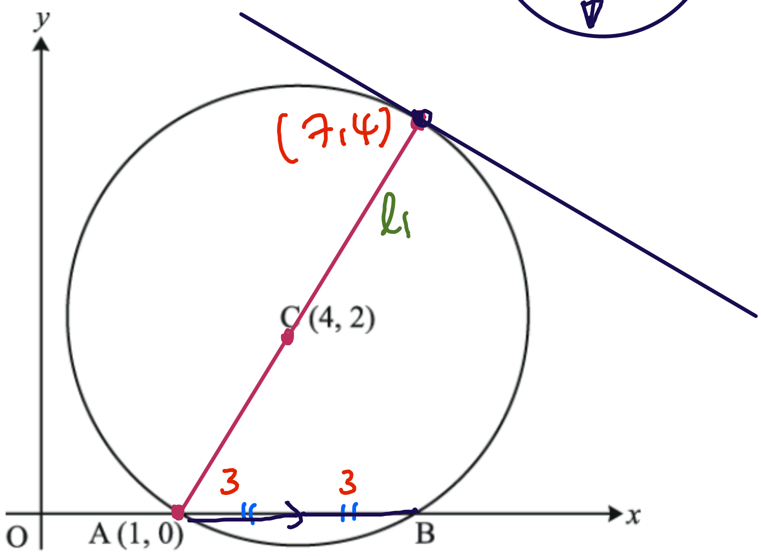


Figure 1 shows a sketch of a circle with centre at $(4, 2)$.
It intersects the x-axis at $A : (1, 0)$ and at $B : (\alpha, \beta)$.

(a) Write down the coordinates of B. **[1]**

$B(7, 0)$

(b) Find the equation of the circle. **[3]**

$(x-4)^2+(y-2)^2=13$

(c) AD is a diameter of the circle. Find the coordinates of D. **[2]**

$(7, 4)$

(d) Find the equation of the tangent to the circle at D.
Give your answer in the form $y = ax + b$. **[3]**

$m_{l_1} = \frac{4-2}{7-4} = \frac{2}{3}$

$m_{\text{tangent}} = \frac{-3}{2}$

$y = \frac{-3}{2}x + c$

$(7, 4)$

$4 = \frac{-3}{2}(7) + c$

$4 + \frac{21}{2} = c$

$\boxed{y = \frac{-3}{2}x + \frac{29}{2}}$

Question 2

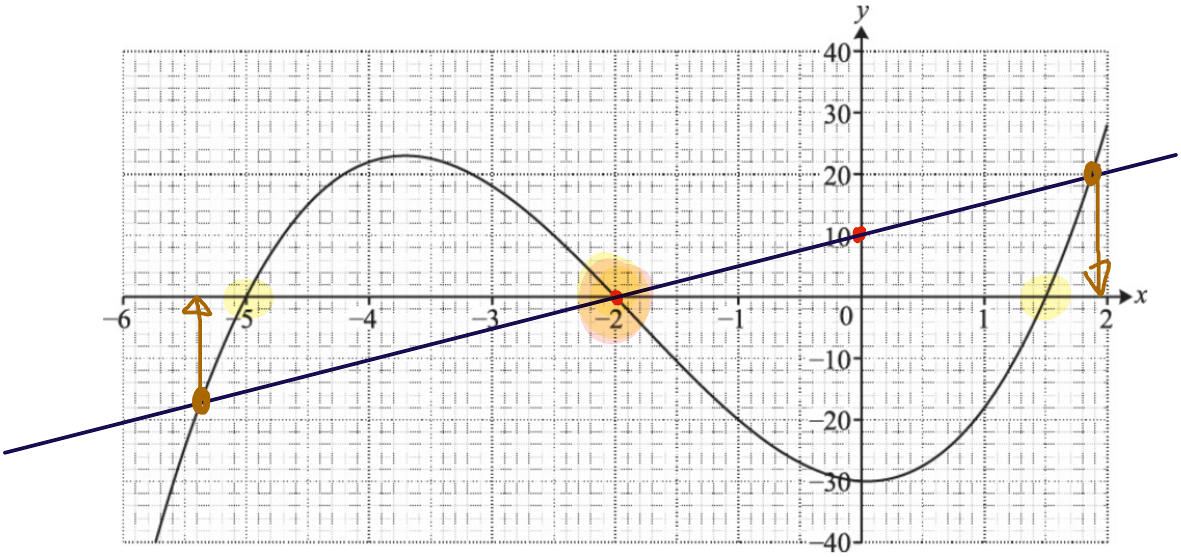


Figure 2 shows the graph of a cubic curve.
It intersects the axes at $(-5, 0)$, $(-2, 0)$, $(\frac{3}{2}, 0)$ and $(0, -30)$.

(a) Use the intersections with both axes to express the equation of the curve in a factorised form. **[2]**

(b) **Hence** show that the equation of the curve may be written as $y = 2x^3 + 11x^2 - x - 30$. **[2]**

(c) Draw the line $y = 5x + 10$ accurately on the graph. Find graphically the x-coordinates of all points of intersection. **[3]**

$X = -5.4, \quad X = -2, \quad X = 1.9$ ✓

(d) By first finding and factorising a ***linear*** factor, show algebraically that the x-coordinates of your answer in ***part (c)*** can be written in the form $(x - k)(2x^2 + 7x - 20) = 0$. **[3]**

a) $y = 2(X+5)(X+2)(X - \frac{3}{2})$

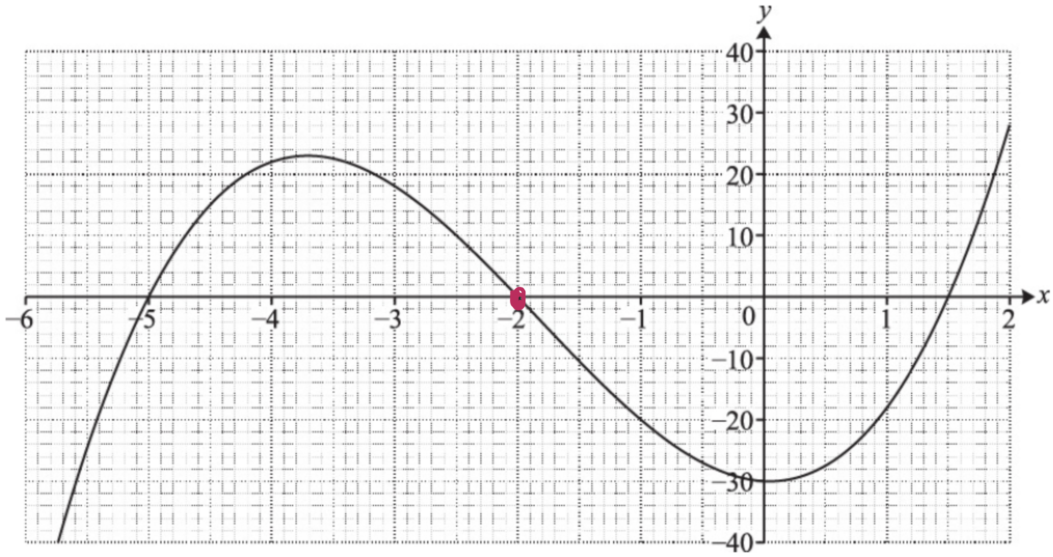
$y = (X+5)(X+2)(2X-3)$

b) $(X^2 + 7X + 10)(2X - 3)$

$= 2X^3 + 14x^2 + 20x - 3x^2 - 21x - 30$

$= 2x^3 + 11x^2 - x - 30$

Question 2



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$2x^3 + 11x^2 - x - 30 = 5x + 10$

$2x^3 + 11x^2 - 6x - 40 = 0$

$x = -2$ is a factor

$\hookrightarrow (x+2)(2x^2 + 7x - 20)$ //

$$
\begin{array}{r}
2x^2+7x-20 \\
x+2 \,\overline{\big)\, 2x^3+11x^2-6x-40} \\
2x^3+4x^2 \\
\hline
7x^2-6x \\
7x^2+14x \\
\hline
-20x-40 \\
-20x-40 \\
\hline
\end{array}
$$